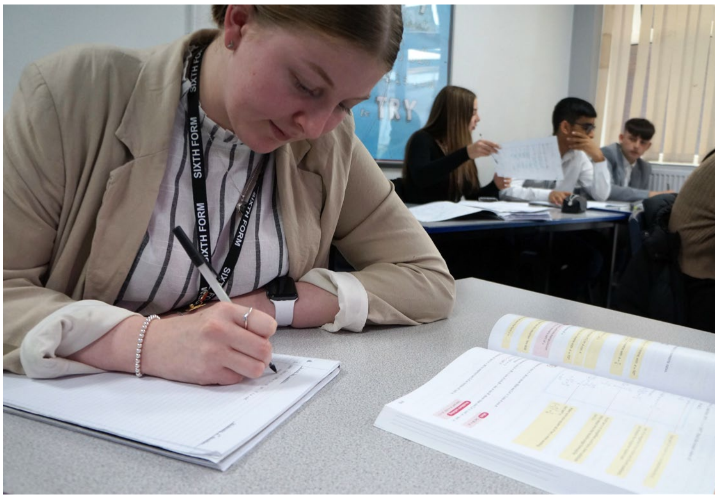
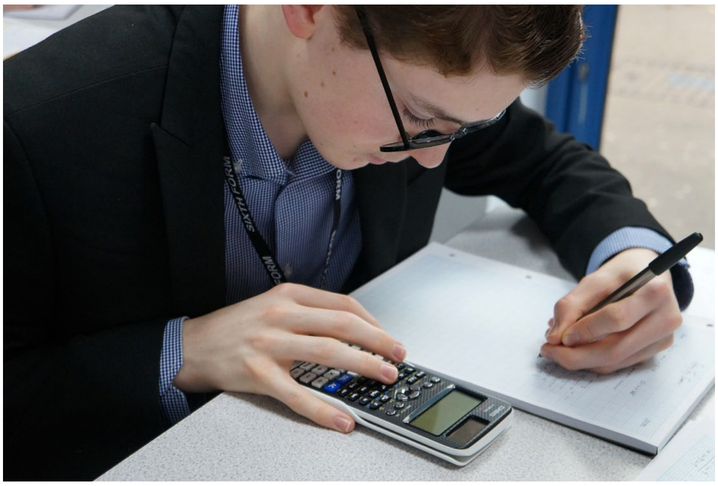
The Friary Sixth Form





A-Level Further Mathematics
Bridging Pack 2023

Course Expectations

Introduction

This pack contains a programme of activities and resources to prepare you to start your A-Level Further Mathematics course in September. It is aimed to be used after you complete your GCSEs, throughout the remainder of the summer term and over the summer holidays to ensure you are ready to start your new course in September.

The course coordinator for this qualification is Mr Thorpe – jthorpe@friary.greywoodmst.co.uk

What we expect from you?

- Excellent attendance/punctuality to lessons
- Correct equipment (see list below)
- Correct uniform – smart business wear and lanyard to be worn at all times
- Meet deadlines
- Contribute positively in lessons

What you can expect from us?

- High quality teaching and learning
- Commitment to you as individuals
- Constant support and guidance
- Weekly after school booster/revisions sessions
- Submitted work will be marked and assessed within 10 days of handing it in

Equipment list

- The recommended calculator for the A-Level Maths course is the, 'CASIO FX-991EX Scientific Calculator'
- A4 folders (x 4 in total for the two years of study)
- A4 note pad (Preferably small squared but lined is acceptable)
- Plastic wallets (for each folder)
- Folder dividers (for each folder)
- Textbooks (provided)
- Pens, pencils, highlighters, rulers

Course Overview

Edexcel – Further Mathematics A-Level

<u>The information provided is taken from the Edexcel specification document</u>

Content and assessment overview

The Pearson Edexcel Level 3 Advanced GCE in Further Mathematics consists of four externally-examined papers. All candidates take Pure Core 1 and Pure Core 2. At the Friary we elect Further Statistics 1 and Decision Mathematics 1 as the optional components.
Students must complete all assessment in May/June in any single year.

Pure Core Mathematics Components

Paper 1: Core Pure Mathematics 1 (*Paper code: 9FM0/01)
Paper 2: Core Pure Mathematics 2 (*Paper code: 9FM0/02)

Each paper is:

1-hour and 30 minutes written examination
25% of the qualification
75 marks

Content overview

Proof
Complex numbers
Matrices
Further algebra and functions
Further calculus
Further vectors
Polar coordinates
Hyperbolic functions
Differential equations

Assessment overview

● Paper 1 and Paper 2 may contain questions on any topics from the Pure Core Mathematics content.
● Students must answer all questions.
● Calculators can be used in the assessment

Optional Component 1

3B: Further Statistics 1

This paper is:

1-hour and 30 minutes written examination
25% of the qualification
75 marks

Content overview

Discrete probability distributions
Poisson & binomial distributions
Geometric and negative binomial distributions
Hypothesis Testing
Central Limit Theorem
Chi Squared Tests
Probability generating functions
Quality of tests

Optional Component 2

3D: Decision Mathematics 1

This paper is:

1-hour and 30 minutes written examination
25% of the qualification
75 marks

Content overview

Algorithms
Graphs and Networks
Algorithms on Graphs
Route Inspection
The Travelling Salesman Problem
Linear Programming
This Simplex Algorithm
Critical Path Analysis

A-Level Further Maths at the Friary

Welcome to A-Level Further Maths at the Friary. Studying Further Maths at A-Level will greatly enhance your understanding and appreciation of the mathematical world while consolidating the fundamentals that are crucial for your Mathematics A-Level.

Our A-Level Further Mathematics team have substantial teaching experience. We also have exam markers in the department so we can give you that extra insight in to exam technique.

The course consists of Pure Mathematics, Further Statistics and Decision. The course is course is carefully structured in order that you might progress through the concepts in a rational order. We aim that the course works logically in parallel with the Single Mathematics A-Level

There may be occasions when you need extra support in particular areas. We carefully monitor your progress to help you recognise these areas and facilitate you in addressing your targets.

If you have any questions about studying Further Mathematics at the Friary then ask one of our teachers or chat to our sixth formers. It's a challenging subject but it's a rewarding experience. The number of students who turn up to our many after school sessions pays testament to the enthusiasm running through the Friary A-Level Maths community.

Tasks

In order to prepare for studying A-Level further mathematics at the Friary School, we have provided the following practice questions .

There are on two key topic areas which we will be studying early in the course which are matrices and complex numbers.

Hopefully the notes and examples will help you complete the questions. I will also be adding tuition videos that will model how you apporach these questions.

If you have any questions about these tasks, please feel free to email me on jthorpe@friary.greywoodmst.co.uk.

Good luck

Mr J Thorpe

Coordinator of A-Level Mathemaitics and Further Mathematics

Matrices - Introduction:

Matrices, and in particular matrix algebra is used in many branches of mathematics. It used extensively in 3D computer graphics, they are also used to manage large amounts of data, or in practical applications such as aviation management control.

Make sure that you set your work out in a clear and logical manner. It should be clearly labeled so that you can find each exercise easily in September.

Definition: A matrix (plural: matrices) is an array of numbers, which is displayed in a rectangular table. The number of rows and columns a matrix has is called the dimension of the matrix. If it has ***n*** rows and ***m*** columns it is said to be an ***n*** by ***m*** matrix or ***n*** x ***m*** matrix. Each number in the matrix is called an element of the matrix.

Examples

$\begin{pmatrix} 4 & 1 & 0 \end{pmatrix}$ is a 1 by 3 matrix

$\begin{pmatrix} -1 & 4 & 6 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ is a 3 by 3 matrix

$\begin{pmatrix} 1 & 0 \\ 9 & 6 \\ -5 & -2 \end{pmatrix}$ is a 3 by 2 matrix

Matrices can be added or subtracted by adding or subtracting the corresponding elements. Matrices can only be added or subtracted if they are the same dimension.

Examples

$$\begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 3+2 & 2+(-3) \\ 0+4 & -2+1 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 1 \\ 2 & 0 \\ 0 & 9 \end{pmatrix} - \begin{pmatrix} 4 & -3 \\ 1 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & -2 \\ 1 & 9 \end{pmatrix}$$

Matrices Practice Section A:

Complete the following questions

1 Describe the dimensions of these matrices.

a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

d $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

e $\begin{pmatrix} 3 & -1 \end{pmatrix}$

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2 For the matrices

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix},$$

find

a $\mathbf{A} + \mathbf{C}$

b $\mathbf{B} - \mathbf{A}$

c $\mathbf{A} + \mathbf{B} - \mathbf{C}$.

3 For the matrices

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} 0 & 1 & -1 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix},$$

find where possible:

a $\mathbf{A} + \mathbf{B}$

b $\mathbf{A} - \mathbf{E}$

c $\mathbf{F} - \mathbf{D} + \mathbf{C}$

d $\mathbf{B} + \mathbf{C}$

e $\mathbf{F} - (\mathbf{D} + \mathbf{C})$

f $\mathbf{A} - \mathbf{F}$

g $\mathbf{C} - (\mathbf{F} - \mathbf{D})$.

4 Given that $\begin{pmatrix} a & 2 \\ -1 & b \end{pmatrix} - \begin{pmatrix} 1 & c \\ d & -2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$, find the values of the constants a, b, c and d.

5 Given that $\begin{pmatrix} 1 & 2 & 0 \\ a & b & c \end{pmatrix} + \begin{pmatrix} a & b & c \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} c & 5 & c \\ c & c & c \end{pmatrix}$, find the values of a, b and c.

6 Given that $\begin{pmatrix} 5 & 3 \\ 0 & -1 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ 2 & 0 \\ 1 & 4 \end{pmatrix}$, find the values of a, b, c, d, e and f.

Scalar Multiplication.

A scalar is a number. Scalar multiplication is when we multiply each element within the matrix by the same number.

Example

$$4\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}4\\0\\8\end{pmatrix}$$

$$\begin{pmatrix}2&1\\1&3\end{pmatrix}+3\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}5&1\\1&6\end{pmatrix}$$

Matrices Practice Section B:

Complete the following questions:-

1 For the matrices $\mathbf{A}=\begin{pmatrix}2&0\\4&-6\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}1\\-1\end{pmatrix}$, find

a $3\mathbf{A}$ **b** $\frac{1}{2}\mathbf{A}$ **c** $2\mathbf{B}$.

2 Find the value of k and the value of x so that $\begin{pmatrix}0&1\\2&0\end{pmatrix}+k\begin{pmatrix}0&2\\-1&0\end{pmatrix}=\begin{pmatrix}0&7\\x&0\end{pmatrix}$.

3 Find the values of a, b, c and d so that $2\begin{pmatrix}a&0\\1&b\end{pmatrix}-3\begin{pmatrix}1&c\\d&-1\end{pmatrix}=\begin{pmatrix}3&3\\-4&-4\end{pmatrix}$.

4 Find the values of a, b, c and d so that $\begin{pmatrix}5&a\\b&0\end{pmatrix}-2\begin{pmatrix}c&2\\1&-1\end{pmatrix}=\begin{pmatrix}9&1\\3&d\end{pmatrix}$.

5 Find the value of k so that $\begin{pmatrix}-3\\k\end{pmatrix}+k\begin{pmatrix}2k\\2k\end{pmatrix}=\begin{pmatrix}k\\6\end{pmatrix}$.

Matrix Multiplication

Two matrices can only be multiplied together if the number of columns in the first matrix is equal to the number of rows in the second. The resulting matrix then has the number of rows from the first matrix and the number of columns from the second.

How matrices are multiplied together is not intuitive. The method is given in general terms for 2 by 2 matrices and examples given for matrices of different dimensions.

Work along the first row of the first matrix while going down the first column of the second matrix, then do the first row & the second column, second row & first column, second row & second column.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

Examples

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2+0 & 3+0 \\ 4+1 & 6+2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+0+3 & 0+4+3 \end{pmatrix} = \begin{pmatrix} 4 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 8 \\ 2 & 8 \end{pmatrix}$$

Important Example

If A = $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ find AB and also BA

Non Commutativity of Matrix Multiplication

You should have found in the example above that the answers AB and BA are not the same.

We say that matrix multiplication is not commutative. (Unlike normal numbers where the order in which we multiply them does not matter, 3 x 2 = 2 x 3).

Therefore, the **order** in which we multiply matrices is important and does matter.

Matrices Practice Section C:

Complete the following questions

1 Given the dimensions of the following matrices:

Matrix	A	B	C	D	E
Dimension	2×2	1×2	1×3	3×2	2×3

Give the dimensions of these matrix products.

a **BA** **b** **DE** **c** **CD**

d **ED** **e** **AE** **f** **DA**

2 Find these products.

a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ -1 & -2 \end{pmatrix}$

3 The matrix $\mathbf{A} = \begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}$ and the matrix $\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

Find

a **AB** **b** $\mathbf{A}^2$

$\mathbf{A}^2$ means $\mathbf{A} \times \mathbf{A}$

4 The matrices **A**, **B** and **C** are given by

$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} -3 & -2 \end{pmatrix}$.

Determine whether or not the following products are possible and find the products of tho that are.

a **AB** **b** **AC** **c** **BC**

d **BA** **e** **CA** **f** **CB**

5 Find in terms of a $\begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}$.

6 Find in terms of x $\begin{pmatrix} 3 & 2 \\ -1 & x \end{pmatrix}\begin{pmatrix} x & -2 \\ 1 & 3 \end{pmatrix}$.

Complex Numbers

The quadratic equation $ax^2 + bx + c = 0$ has solutions given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If the expression under the square root is negative, there are no real solutions.

Links For the equation $ax^2 + bx + c = 0$, the **discriminant** is $b^2 - 4ac$.
- If $b^2 - 4ac > 0$, there are two distinct real roots.
- If $b^2 - 4ac = 0$, there are two equal real roots.
- If $b^2 - 4ac < 0$, there are no real roots.

← Pure Year 1, Section 2.5

You can find solutions to the equation in all cases by extending the number system to include $\sqrt{-1}$. Since there is no real number that squares to produce -1, the number $\sqrt{-1}$ is called an **imaginary number**, and is represented using the letter **i**. Sums of real and imaginary numbers, for example $3 + 2i$, are known as **complex numbers**.

- $\mathbf{i} = \sqrt{-1}$
- **An imaginary number is a number of the form bi, where $b \in \mathbb{R}$.**
- **A complex number is written in the form $a + bi$, where $a, b \in \mathbb{R}$.**

Notation The set of all complex numbers is written as $\mathbb{C}$.
For the complex number $z = a + bi$:
- $\mathrm{Re}(z) = a$ is the real part
- $\mathrm{Im}(z) = b$ is the imaginary part

Example 1

Write each of the following in terms of i.

a $\sqrt{-36}$ **b** $\sqrt{-28}$

a $\sqrt{-36} = \sqrt{36 \times (-1)} = \sqrt{36}\sqrt{-1} = 6i$

b $\sqrt{-28} = \sqrt{28 \times (-1)} = \sqrt{28}\sqrt{-1}$
$= \sqrt{4}\sqrt{7}\sqrt{-1} = (2\sqrt{7})i$

You can use the rules of surds to manipulate imaginary numbers.

Watch out An alternative way of writing $(2\sqrt{7})i$ is $2i\sqrt{7}$. Avoid writing $2\sqrt{7}i$ as this can easily be confused with $2\sqrt{7i}$.

In a complex number, the real part and the imaginary part cannot be combined to form a single term.

- **Complex numbers can be added or subtracted by adding or subtracting their real parts and adding or subtracting their imaginary parts.**
- **You can multiply a real number by a complex number by multiplying out the brackets in the usual way.**

Example 2

Simplify each of the following, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

a $(2 + 5i) + (7 + 3i)$ **b** $(2 - 5i) - (5 - 11i)$ **c** $2(5 - 8i)$ **d** $\frac{10 + 6i}{2}$

a $(2 + 5i) + (7 + 3i) = (2 + 7) + (5 + 3)i$
$= 9 + 8i$

Add the real parts and add the imaginary parts.

b $(2 - 5i) - (5 - 11i) = (2 - 5) + (-5 - (-11))i$
$= -3 + 6i$

Subtract the real parts and subtract the imaginary parts.

c $2(5 - 8i) = (2 \times 5) - (2 \times 8)i = 10 - 16i$

$2(5 - 8i)$ can also be written as $(5 - 8i) + (5 - 8i)$.

d $\frac{10 + 6i}{2} = \frac{10}{2} + \frac{6}{2}i = 5 + 3i$

First separate into real and imaginary parts.

Example 4

Solve the equation $z^2 + 6z + 25 = 0$.

Method 1 (Completing the square)

$z^2 + 6z = (z + 3)^2 - 9$

Because $(z+3)^2 = (z+3)(z+3) = z^2 + 6z + 9$

$z^2 + 6z + 25 = (z + 3)^2 - 9 + 25 = (z + 3)^2 + 16$

$(z + 3)^2 + 16 = 0$

$(z + 3)^2 = -16$

$z + 3 = \pm\sqrt{-16} = \pm 4\text{i}$

$\sqrt{-16} = \sqrt{16 \times (-1)} = \sqrt{16}\sqrt{-1} = 4\text{i}$

$z = -3 \pm 4\text{i}$

$z = -3 + 4\text{i}, \quad z = -3 - 4\text{i}$

You can use your calculator to find the complex roots of a quadratic equation like this one.

Method 2 (Quadratic formula)

$z = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 25}}{2}$

Using $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm \sqrt{-64}}{2}$

$z = \dfrac{-6 \pm 8\text{i}}{2} = -3 \pm 4\text{i}$

$\sqrt{-64} = \sqrt{64 \times (-1)} = \sqrt{64}\sqrt{-1} = 8\text{i}$

$z = -3 + 4\text{i}, \quad z = -3 - 4\text{i}$

Complex Numbers Practice Section A:

3 Simplify, giving your answers in the form $a + b\text{i}$, where $a, b \in \mathbb{R}$.

a $2(7 + 2\text{i})$ b $3(8 - 4\text{i})$

c $2(3 + \text{i}) + 3(2 + \text{i})$ d $5(4 + 3\text{i}) - 4(-1 + 2\text{i})$

e $\dfrac{6 - 4\text{i}}{2}$ f $\dfrac{15 + 25\text{i}}{5}$

g $\dfrac{9 + 11\text{i}}{3}$ h $\dfrac{-8 + 3\text{i}}{4} - \dfrac{7 - 2\text{i}}{2}$

(P) 4 Write in the form $a + b\text{i}$, where a and b are simplified surds.

a $\dfrac{4 - 2\text{i}}{\sqrt{2}}$ b $\dfrac{2 - 6\text{i}}{1 + \sqrt{3}}$

5 Given that $z = 7 - 6\text{i}$ and $w = 7 + 6\text{i}$, find, in the form $a + b\text{i}$, where $a, b \in \mathbb{R}$:

a $z - w$ b $w + z$

Notation Complex numbers are often represented by the letter z or the letter w.

(E) 6 Given that $z_1 = a + 9\text{i}$, $z_2 = -3 + b\text{i}$ and $z_2 - z_1 = 7 + 2\text{i}$, find a and b where $a, b \in \mathbb{R}$. **(2 marks)**

(P) 7 Given that $z_1 = 4 + \text{i}$ and $z_2 = 7 - 3\text{i}$, find, in the form $a + b\text{i}$, where $a, b \in \mathbb{R}$:

a $z_1 - z_2$ b $4z_2$ c $2z_1 + 5z_2$

(P) 8 Given that $z = a + b\text{i}$ and $w = a - b\text{i}$, $a, b \in \mathbb{R}$, show that:

a $z + w$ is always real b $z - w$ is always imaginary

You can use complex numbers to find solutions to any quadratic equation with real coefficients.

- **If $b^2 - 4ac < 0$ then the quadratic equation $ax^2 + bx + c = 0$ has two distinct complex roots, neither of which are real.**

Solve the equation $z^2 + 6z + 25 = 0$.

Method 1 (Completing the square)

$z^2 + 6z = (z + 3)^2 - 9$

Because $(z + 3)^2 = (z + 3)(z + 3) = z^2 + 6z + 9$

$z^2 + 6z + 25 = (z + 3)^2 - 9 + 25 = (z + 3)^2 + 16$

$(z + 3)^2 + 16 = 0$

$(z + 3)^2 = -16$

$z + 3 = \pm\sqrt{-16} = \pm 4i$

$\sqrt{-16} = \sqrt{16 \times (-1)} = \sqrt{16}\sqrt{-1} = 4i$

$z = -3 \pm 4i$

$z = -3 + 4i, \quad z = -3 - 4i$

You can use your calculator to find the complex roots of a quadratic equation like this one.

Method 2 (Quadratic formula)

$z = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 25}}{2}$

Using $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm \sqrt{-64}}{2}$

$z = \dfrac{-6 \pm 8i}{2} = -3 \pm 4i$

$\sqrt{-64} = \sqrt{64 \times (-1)} = \sqrt{64}\sqrt{-1} = 8i$

$z = -3 + 4i, \quad z = -3 - 4i$

Complex Numbers Practice Section B:

Do not use your calculator in this exercise:

3 Solve each of the following equations. Write your answers in the form $a \pm bi$.

a $z^2 + 2z + 5 = 0$ **b** $z^2 - 2z + 10 = 0$ **c** $z^2 + 4z + 29 = 0$

d $z^2 + 10z + 26 = 0$ **e** $z^2 + 5z + 25 = 0$ **f** $z^2 + 3z + 5 = 0$

4 Solve each of the following equations. Write your answers in the form $a \pm bi$.

a $2z^2 + 5z + 4 = 0$ **b** $7z^2 - 3z + 3 = 0$ **c** $5z^2 - z + 3 = 0$

5 The solutions to the quadratic equation $z^2 - 8z + 21 = 0$ are z_1 and z_2.
Find z_1 and z_2, giving each in the form $a \pm i\sqrt{b}$.

6 The equation $z^2 + bz + 11 = 0$, where $b \in \mathbb{R}$, has distinct non-real complex roots.
Find the range of possible values of b.

Glossary

The following terms are used in questions and assessments. It is essential that you familiarise yourself with them.

Exact - An exact answer is one where numbers ARE NOT given in rounded form. The answer will often contain an irrational number such as $\sqrt{3}$, e or π and these numbers should be given in that form. Rigorous (exact) working is expected in the answer to the question.

Hence - When a question uses the word 'hence' it is an indication that the next step should be based on what has gone before. You should start from this statement. Where the phrase "Hence or otherwise" is used, this indicates that whilst the previous work could form the starting point of the solution, learners may be aware of, and could use, an equally valid alternate method.

Show that - Show a result is true. Because you're given the result, your explanation has to be sufficiently detailed to cover every step of your working.

Prove - Provide a formal mathematical argument to demonstrate validity

Verify - Substitute given values to demonstrate the truth of a statement.

Sketch - Draw a diagram, not necessarily to scale, showing the main features of a curve.

Find, solve, calculate - While working may be necessary to answer the question, no justification needs to be given for any results found.

Show detailed reasoning - Give a solution that leads to a conclusion showing a detailed and complete analytical method. Your solution should contain sufficient detail to allow the line of your argument to be followed. This is not a restriction on use of a calculator when tackling the question.

Determine - Justification should be given for any results found, including working where appropriate.

Draw - Draw to an accuracy appropriate to the problem. You are being asked to make a sensible judgement about this.

Additional Reading

As a student who is choosing to study Mathematics at A Level, it is logical to assume that you have an interest in the subject.

With that said, the following books may be of interest to you. These are books that are not directly linked to the course but allow you to further your mathematical intrigue and understanding.

50 Mathematical Ideas You Really Need to Know (Tony Crilly)
Alex's Adventures in Numberland (Alex Bellos)
Cabinet of Mathematical Curiosities (Ian Stewart)
The Calculus Wars **(**Jason Socrates Bardi)
The Code Book (Simon Singh)
The Curious Incident of the Dog in the Night-time by Mark Haddon
How Many Socks Make a Pair?: Surprisingly Interesting Maths (Rob Eastway)
Hello World: How to be Human in the Age of the Machine (Hannah Fry)
Humble Pi: A Comedy of Maths Errors (Matt Parker)
The Life-Changing Magic of Numbers (Bobby Seagull)
The Num8er My5teries (Marcus du Sautoy)

Supporting Resources

- You will be provided with the relevant text books for the Edexcel Further Mathematics A-Level published by Pearson
- The recommended calculator for the A-Level Maths course is the, 'CASIO FX-991EX Scientific Calculator'
- There are supplementary texts available. We recommend the CGP revision guides and workbooks. There will be opportunities to purchase these through the school.